Mountain goats in the Budapest Zoo (photograph by Istvan Hargittai) displaying gradual size and age changes. They can be considered to be a segment of an “infinite” succession of *similarity symmetry* (see in Figure 16a in the following article).

The Universality of the Symmetry Concept[a]

István Hargittai and Magdolna Hargittai

Abstract

The notion of symmetry brings together beauty and usefulness, science and economy, mathematics and human relations. This presentation demonstrates the breadth and versatility of the symmetry concept. There are no symmetries specific to various disciplines, yet there are differences in emphasis in applications of the concept. The sciences, humanities and arts have gradually drifted apart; symmetry can provide a connecting link among them. The symmetry concept may be broadened to include harmony and proportion, constituents of symmetry often present in architectural composition. The symmetries considered here are point group, chiral, space group, and translational. While mathematical symmetry is exact and rigorous, the symmetry we encounter in everyday life is much more relaxed. The broad interpretation of the symmetry concept, coming close to blending fact and fantasy, may help scientists recognize trends, changes, and patterns.

Introduction

The notion of symmetry brings together beauty and usefulness, science and economy, mathematics and music, architecture and human relations, and much more, as has been shown recently with many examples (Hargittai 1986, 1989; Hargittai and Hargittai 1995, 1996). There is a lot of symmetry, for example, in Béla Bartók's music. It is not known, however, whether he consciously applied symmetry or was simply led intuitively to the golden ratio so often present in his music. Bartók himself always refused to discuss the technicalities of his composing and stated merely "We create after Nature." Another unanswerable question is how these symmetries contribute to the appeal of Bartók's music, and how much of this appeal originates from our innate sensitivity to symmetry. This question might be equally asked of symmetries in architectural composition.

The present chapter takes a broad view of the symmetry concept. It demonstrates its breadth and versatility. There are no distinctly different specific symmetries in various disciplines, yet there are discernible differences in emphasis of the application of this concept in different fields. This emphasis changes with time as well. For example, there is a marked emphasis on the presence of symmetry in chemistry, in contrast to physics

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where the importance of broken symmetries has been stressed during the past decades. Generally though the symmetry concept unites rather than divides the different branches of science, and even helps bridge the gap between what C.P. Snow called "two cultures." Sciences, the humanities, and the arts have all drifted apart over the years and symmetry can provide a connecting link among them. Its benefits are available to us if we free ourselves from the confinements of *geometrical symmetry*.

Everything is rigorous in geometrical symmetry. According to one definition, "symmetry is the property of geometrical figures to repeat their parts" (Shubnikov 1951). Another definition says that "a figure is symmetrical if there is a congruent transformation which leaves it unchanged as a whole, merely permuting its component elements" (Coxeter 1973). In the geometrical sense, symmetry is either present or it is absent. Any question regarding symmetry has a restricted *yes/no* alternative. For the real, material world, however, degrees of symmetry and even gradual symmetry is feasible and applicable. Beyond geometrical definitions there is another, broader meaning to symmetry—one that relates to *harmony* and *proportion*, and ultimately to *beauty*. This aspect involves feeling and subjective judgment and, as a result, is especially difficult to describe in technical terms.

Simple considerations are indispensable in classifying different kinds of symmetry. There are two large classes of symmetry, *point groups* and *space groups*. For point group symmetries there is at least one special point in the object or pattern that differs from all the others. In contrast to this, in space groups, there is no such special point. There are also some terms that are useful in the description of different types of symmetry. Thus, the action that characterizes a particular type of symmetry is called a *symmetry operation*. The tool whereby the operation is performed is called a *symmetry element*.

Point Group Symmetry

The simplest kind of point-group symmetry is *bilateral symmetry*. Bilateral symmetry is present when two halves of the whole are each other's mirror images (Fig. 40.1). This is the most common symmetry and the every-day usage of the term "symmetry" refers to this meaning. The symmetry element is a *mirror plane*, also called a *symmetry plane* or a *reflection plane*. The symmetry operation is *reflection*. Applying a mirror plane to either of the two halves of an object with bilateral symmetry recreates the whole object. Bilateral symmetry is probably the most common symmetry in architecture as well, from simple buildings to larger assemblies (Fig. 40.2a, b).

Fig. 1 The orchid has bilateral symmetry. Photo: authors

Fig. 2 (**a**) The whole assembly of the Blue Mosque in Istanbul, Turkey, has bilateral symmetry. (**b**) The design of St. Peter's Square in Vatican City also shows bilateral symmetry. Photo: authors

Another kind of point-group symmetry is *rotational symmetry* (Fig. 40.3). It is present when, by rotating an object around its axis, it appears in the same position two or more times during a full revolution. *Rotation* is the symmetry operation and the *axis of rotation* is the symmetry element. Rotational symmetry may be twofold, threefold, fourfold, etc. It is common that reflection and rotation appear together. The presence of some symmetry elements may generate others and vice versa. If we look at the Eiffel tower from below (Fig. 40.4) we have twice two orthogonal reflection planes which generate a fourfold rotation. The cupolas of many state capitols and other important buildings have reflectional and rotational symmetry together (Fig. 40.5).

Fig. 3 This hubcap has sevenfold rotational symmetry. Photo: authors

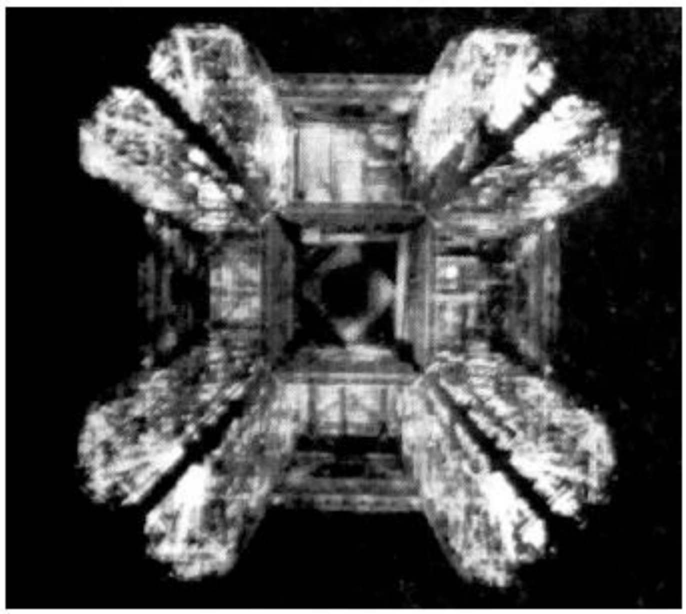

Fig. 4 The Eiffel Tower from below. It shows both reflections and rotational symmetry. Photo: authors

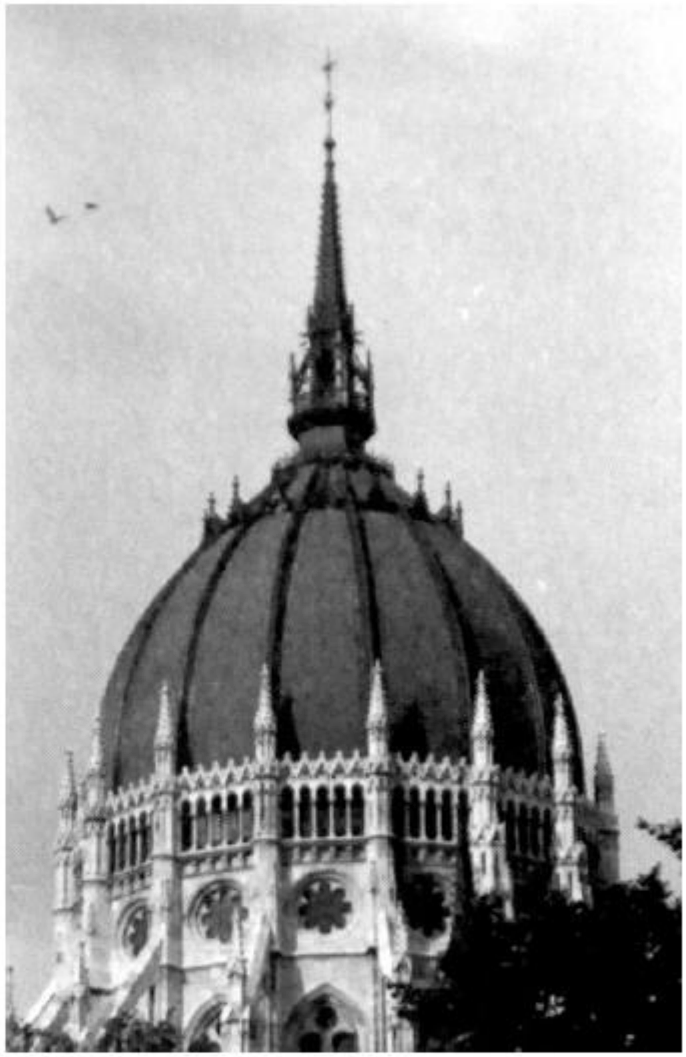

Fig. 5 The cupola of the Hungarian Parliament with both reflectional and rotational symmetry. Photo: authors

The regular polygons, so basic in architectural design, also have both rotational and reflectional symmetry. Best seen when viewed from above, many buildings have outlines of a regular polygon (Fig. 40.6). The regular polyhedra, also called Platonic solids, all have equal regular polygons as their faces. As H.S.M. Coxeter, professor of mathematics at the University of Toronto, remarked, “the chief reason for studying regular polyhedra is still the same as in the times of the Pythagoreans.” Namely, that their symmetrical shapes appeal to one’s artistic sense. There are other highly symmetrical polyhedra, called Archimedian polyhedra, whose faces are also regular polygons but not identical ones. Buckminster Fuller’s geodesic dome is composed of lightweight bars forming regular polygons. His geodesic dome at the Montreal expo (Fig. 40.7) inspired some chemists who saw that the structure of a newly discovered substance may be the truncated icosahedron. This molecule, C_{60}, called buckminsterfullerene (Fig. 40.8) is characterized, among others, by six axes of fivefold rotation (Hargittai and Hargittai 1994: 100–101). Experimentally discovered in 1985, its great relative stability was predicted already in 1970, based solely on symmetry considerations.

Fig. 6 The outline of the Pentagon in Washington, D.C. with its regular *pentagonal shape*. Photo: authors

Fig. 7 Buckminster Fuller’s Geodesic Dome at the Montreal Expo. Photo: authors

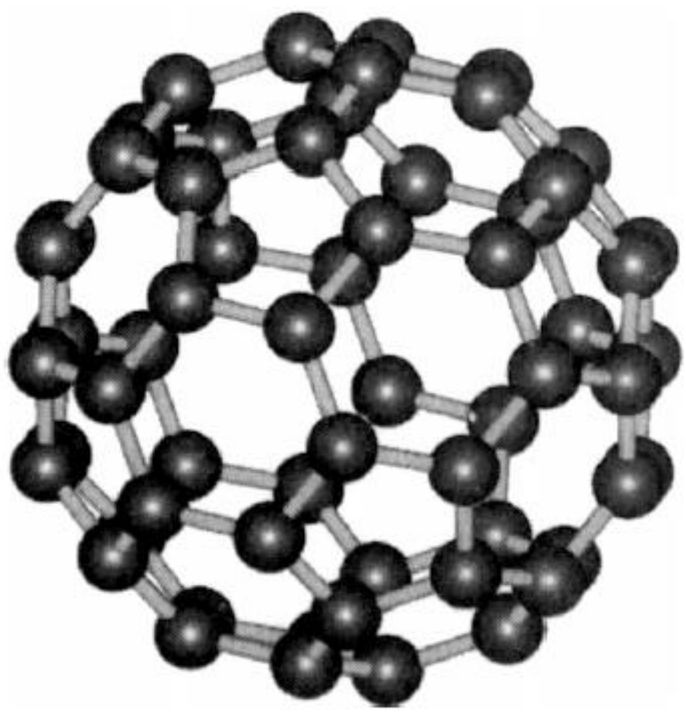

Fig. 8 C_{60}, the buckminsterfullerene molecule. Image: authors

Chirality

A special kind of symmetry relationship is when two objects are related by mirror reflection and the two objects cannot be superposed. Our hands are an excellent example, and the term *chiral* derives from the Greek word for hand. Chiral objects have senses and following the hand analogy they are left-handed (L) and right-handed (D). The simplest chiral molecule is a methane derivative in which three of the four hydrogens are replaced by three different atoms, such as, for example, fluorine (F), chlorine (Cl), and bromine (Br). There may then be a left-handed C(HFClBr) and a right-handed C(HFClBr) molecule which will be each other's mirror images but won't be superposable (Fig. 40.9). A chiral object and its mirror image are called each other's *enantiomorphs*.

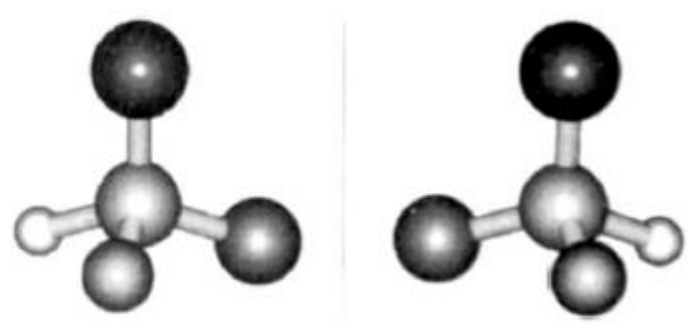

Fig. 9 A chiral pair of molecules. Image: authors

The two chiral molecules look the same in every detail; only their senses are different. The distinctions between the twins of a chiral pair have literally vital significance. Only l-amino acids are present in natural proteins and only d-nucleotides are present in natural nucleic acids. This happens in spite of the fact that the energy of both enantiomers is equal and their formation has equal probability in an achiral environment. However, only one of the two occurs in nature, and the particular enantiomers involved in life processes are the same in humans, animals, plants, and microorganisms. The origin of this phenomenon is a great puzzle.

Once a chiral molecule happens to be in a chiral environment, the two chiral isomers will be behaving differently. This different behaviour is manifested sometimes in very dramatic ways. In some cases one isomer is sweet, the other is bitter. In some other cases the drug molecule has an "evil twin." A tragic example was the thalidomide case in the 1950s in Europe, in which the right-handed molecule cured morning sickness and the left-handed one caused birth defects. Other examples include one enantiomer of ethambutol fighting tuberculosis with its evil twin causing blindness, and one enantiomer of naproxen reducing arthritic inflammation with its evil twin poisoning the liver. Ibuprofen is a lucky case in which the twin of the chiral form that provides the curing is converted to the beneficial version by the body.

Even when the twin is harmless, it represents waste and a potential pollutant. Thus, a lot of efforts are directed toward producing enantiomerically pure drugs and pesticides. One of the fascinating possibilities is to produce sweets from chiral sugars of the enantiomer that would not be capable of contributing to obesity yet would retain the taste of the other enantiomer.

Chiral symmetry is also frequently found in architectural design either in two- or in three dimensions, as illustrated by Fig. 40.10.

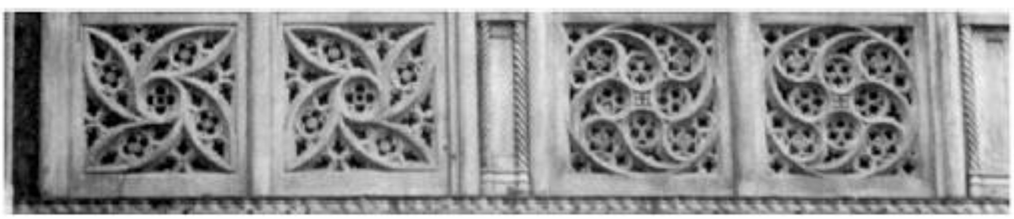

Fig. 10 Chiral rosettes on a building in Bern, Switzerland. Photo: authors

Space Group Symmetry

A different kind of symmetry can be created by simple *repetition* of a basic motif leading us to *space-group symmetries*. The most economical growth and expansion patterns are described by space-groups symmetries. There are three basic cases of space groups, depending on whether the basic motif extends periodically in one direction only, or in two, or finally, in three. These three cases are described by the so-called *one-dimensional, two-dimensional*, and *three-dimensional* space groups.

Border decorations are examples of one-dimensional space groups. In border decorations a pattern can be generated simply by repeating a motif at equal intervals. This is *translational* symmetry. The symmetry element is *constant translation;* the operation is the *translation* itself. The resulting pattern shows periodicity in one direction. Repetition can be achieved by a simple shift in one direction as can be seen very often in the rows of columns of grandiose buildings (Fig. 40.11) or in the ancient aqueducts of the Romans. Fences are typical examples of one-dimensional space groups (Fig. 40.12), the ease and economy of using the same elements repeatedly makes this obvious. Repetition can also be achieved in other ways, such as by reflection, rotation (Fig. 40.13), or *glide- reflection*. Glide-reflection is another new element that does not occur in point-group symmetries. It means the consecutive application of translation and horizontal reflection. When we walk in wet sand along a straight line we leave behind a pattern of footprints whose symmetry is described by glide-reflection. There is a total of seven possibilities for generating one-dimensional space-group symmetries.

Fig. 11 Colonnade on St. Peter's square in Vatican City. Photo: authors

Fig. 12 Repeating pattern of a fence in the Topkapi Palace in Istanbul, Turkey. Photo: authors

Fig. 13 Another illustration for one-dimensional space groups: the units turn 90° at every translation in this chain. Photo: authors

Helices and spirals have also one-dimensional space-group symmetries although their bodies may extend to three dimensions (Hargittai and Pickover 1992). *Helical symmetry* is created by a constant amount of translation accompanied by a constant amount of rotation. In *spiral symmetry*, again, translation is accompanied by rotation but the amount of translation and rotation changes gradually and regularly. An extended spiral staircase has helical symmetry. Well-ordered biological macromolecules also have helical symmetry. Helices are always three-dimensional whereas there are spirals that extend in two dimensions only. Occurrences of spirals may be as diverse as chemical waves and galaxies and snails. Spirals and helices have also been used in various ways in architecture, from ancient times to the present, as Trajan's column in the Forum Romanum (Fig. 40.14) and the spiral ramp of Frank Lloyd Wright's Guggenheim Museum in New York indicate.

Fig. 14 Spiral symmetry of Trajan's column in the Forum Romanum in ancient Rome. Photo: authors

Another beautiful example of spiral symmetry is the scattered leaf arrangement around the stems of plants, called *phyllotaxis*. Numbers of the Fibonacci series (1, 1, 2, 3, 5, 8, 13, 21, …—each new element is the sum of the two previous elements) characterize the ratios defining the occurrence of every consecutive new leaf in scattered leaf arrangements. Thus, for example, there is a new leaf at each 3/8 parts of the circumference of the stem as we move along the stem in one of the characteristic cases. The pineapple (Fig. 40.15) displays a pattern of spirals that can be thought of as if it were a result of compressed phyllotaxis. Such ratios when involving very large numbers approximate an important irrational number, 0.381966…, expressing the so-called *golden ratio*. The golden ratio is created by the golden section in which a given length is divided such that the ratio of the longer part to the whole is the same as the ratio of the shorter part to the longer part. If the whole is 1.00, the lengths of the longer and shorter parts will be 0.618 and 0.382, respectively. This may be the single most important proportion in architecture and in artistic expression. Its relationship to phyllotaxis may have inspired Leonardo da Vinci's description of the scattered leaf arrangement as "more beautiful, more simple, or more direct" than anything humans could devise (Leonardo da Vinci 1939).

Fig. 15 The pineapple displays a pattern of spirals that can be thought of as if it were a result of compressed phyllotaxis. Photo: authors

Spiral symmetry can also be considered as belonging to the broad concept of *similarity symmetry*. Here pattern generation always involves an increment of a characteristic property (Fig. 40.16).

Fig. 16 (**a**) Similarity symmetry, the increments being the change in size or the change in age. (**b**) An architectural example of similarity symmetry where the increment is the change in size of the units of the church-tower in London, England. Photo: authors

With two-dimensional space-groups, there is a total of *17* ways to generate different patterns. It is a special case when the planar network covers the plane without gaps and overlaps. Of the regular polygons, only the equilateral triangle, the square, and the regular hexagon are capable of covering the plane without gaps and overlaps. For arbitrary shapes though, there are infinite possibilities. M.C. Escher’s periodic drawings and the wall decorations in the Alhambra of Granada, Spain (Fig. 40.17) are famous examples. The façades of buildings, especially those of modern skyscrapers often display symmetries in two dimensions (Fig. 40.18).

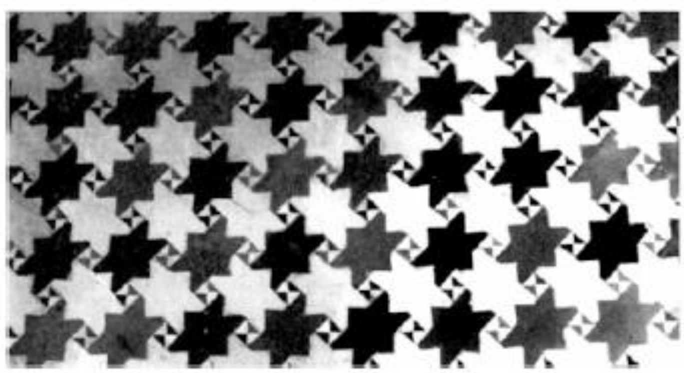

Fig. 17 Two-dimensional space group: decoration from the Alhambra Granada, Spain. Photo: authors

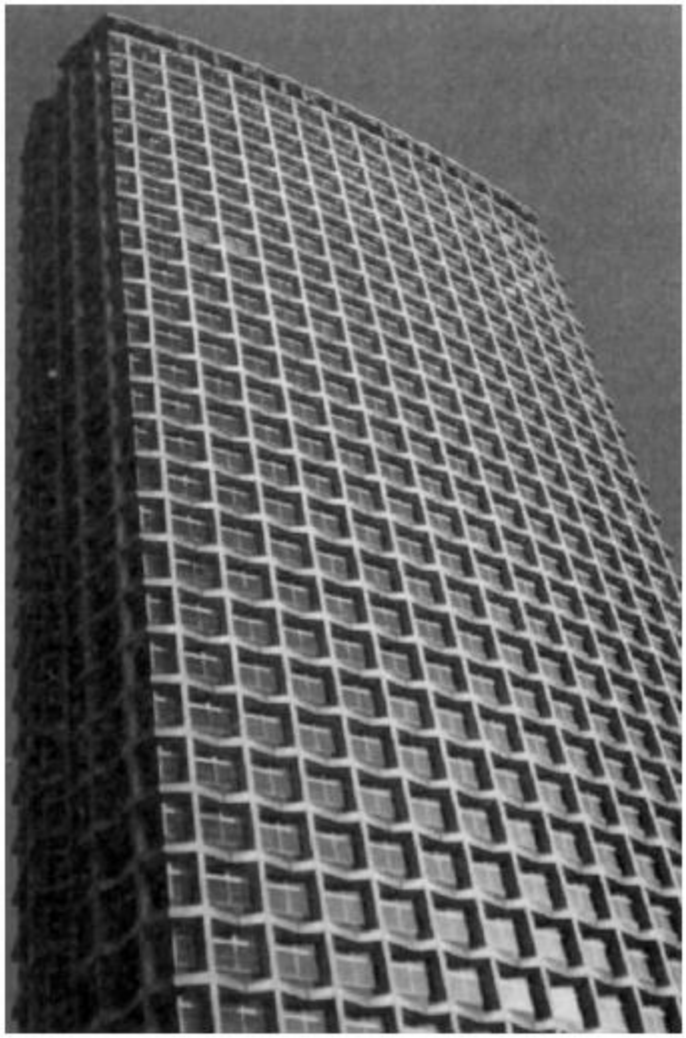

Fig. 18 The façades of modern skyscrapers are typical examples of repetitions in two dimensions. Photo: authors

Space utilization by periodic arrangements seems to be the underlying principle of the occurrence of three-dimensional space-group symmetries. This is a common arrangement of the building elements in *crystals*. The packing of spheres was first considered as the key to the internal structure of crystals by Johannes Kepler. As he was looking at the exquisitely beautiful hexagonal snowflakes, he made drawings of sphere packing, similar to a pyramid of canon balls (Fig. 40.19).

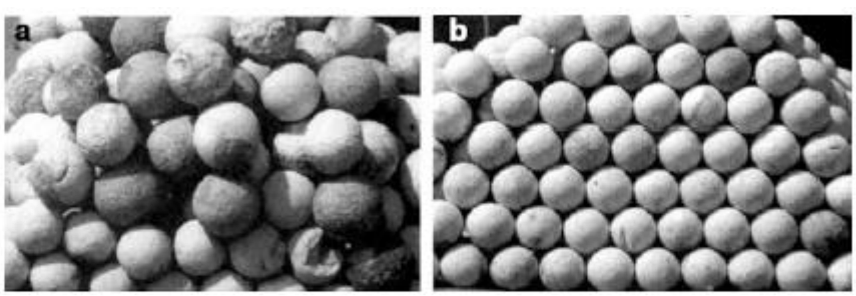

Fig. 19 Random arrangement of canon balls provides much poorer space utilization than their regular arrangement. Photo: authors

There are restrictions for the regular and periodic structures, such as the nonavailability of fivefold symmetry in generating them. This can be understood easily when we find it impossible to cover the plane without gaps or overlaps with equal-size regular pentagons.

Crystals are advantageous for the determination of the structure of molecules. The great success of X-ray crystallography may have diverted attention from structures of lesser symmetry though of not necessarily lesser importance. The discovery of quasiperiodic crystals [in short, quasicrystals (Hargittai 1990)] by the Israeli scientist Dan Shechtman in 1982 has by now persuaded many scientists that their view of crystals is unnecessary narrow. David Mermin compared abandoning the traditional classification scheme of crystallography, based on periodicity, to abandoning the Ptolemaic view in astronomy, and likened changing it to a new foundation to astronomy's adopting the Copernican view (Mermin 1992).

Recently, even such descriptive fields of biology as zoology have displayed a growing activity in symmetry matters. Not surprisingly, the role of external symmetry is being recognized as decisive in mate selection. Empirical evidence supports the notion relating "animal beauty" to the symmetry of outlook. The degree of left-and-right correspondence of the wings seems to correlate with hormone and pheromone production (Angier 1994: C1).

In view of the fundamental importance of the symmetry concept, it is surprising that even very basic discoveries about it were left to be made in this century. When P.A.M. Dirac was asked about Einstein's most important contributions to physics, he singled out Einstein's "introduction of the concept that space and time are symmetrical" (Yang 1991: 11). An important step was Emmy Noether's recognition that symmetry and conservation are connected. Indeed, the idea that the great conservation laws of physics, like the conservation of energy and momentum, are related to symmetry opened up a wholly new way of thinking for scientists. Realizing that Nature included continuous symmetry in her design physicists started to look for new connections.

It was Dirac who had the prescience to write already in 1949, that "I do not believe that there is any need for physical laws to be invariant under reflections" (Dirac 1949). Yet, even most physicists were surprised by the discovery of the nonconservation of parity in 1957 that brought the Nobel prize in physics to T.D. Lee and C.N. Yang. C.P. Snow called this discovery one of the most astonishing in the whole history of science. Since then broken symmetries have been receiving increasing attention.

There seems to be a difference in approach and emphasis between physicists and chemists in viewing symmetry. It may even be related to the ancient Greek philosophers, stressing the importance of continuum by Aristotle, and of the discreet, by Lucretius and Democritos. From the point of view of continuum, even the ideal crystal may be discussed in terms of broken symmetries. On the other hand, the chemist's approach is succinctly symbolized by Democritos' statement: "Nothing exists except atoms and empty space; everything else is opinion."

Of course, the way symmetry is looked at can vary a great deal. While mathematical symmetry is exact and rigorous, the symmetry we encounter in everyday life is much more relaxed. The vague and fuzzy interpretation of the symmetry concept may also aid scientists to recognize trends, characteristic changes, and patterns. This is getting close to blending fact and fantasy. As Arthur Koestler expressed it, “artists treat facts as stimuli for the imagination, while scientists use their imagination to coordinate facts” (Koestler 1949).

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