Lion and
Buckyball

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Cover of the Summer 1995 issue of *The Mathematical Intelligencer* from I. Hargittai, "Fullerene Geometry under the Lion's Paw." © 1995, Springer.

Fullerene Geometry under the Lion's Paw[a]

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" . . . the spherical is the form of all forms most perfect, having need of no articulation; and the spherical is the form of greatest volumetric capacity, best able to contain and circumscribe all else; and all the separated parts of the world—I mean the sun, the moon, and the stars--are observed to have spherical form; and all things tend to limit themselves under this form--as appears in drops of water and other liquids--whenever of themselves they tend to limit themselves. So no one may doubt that the spherical is the form of the world, the divine body."

From Copernicus, *De Revolutionibus Orbium Caelestium,* 1543. Heaven, in fact, is often depicted as a sphere in sculptures. The sphere may also be a representation of the Globe, and it is said to symbolize power as well.
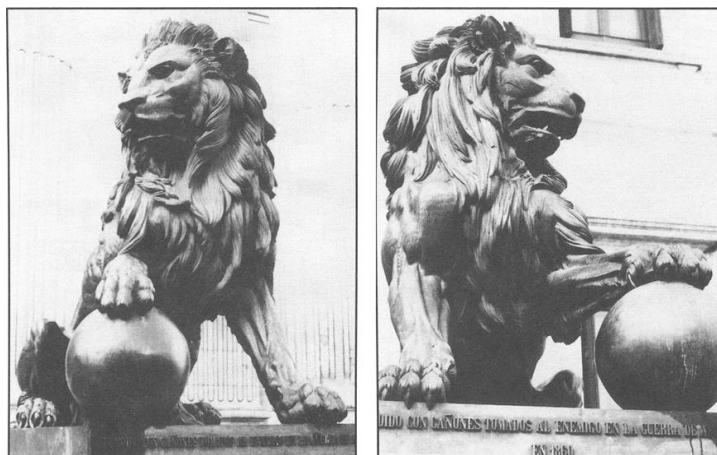

Figure 1. Two lions in front of the Chamber of Deputies *(Congreso de los Diputados),* Madrid, Spain

Two lions stand guard in front of the Spanish Chamber of Deputies *(Congreso de los Diputados).* One of them has a sphere under the right paw and the other under the left paw (Figure 1). The surfaces of these spheres are smooth, without any decoration.

It has been common practice in China to have lion sculptures in front of important (and not so important) buildings. These lions also appear in pairs. The female has a baby lion under the left paw and the male has a sphere under the right paw. The female lion is apparently teasing the baby lion while the sphere under the male's paw is said to represent a ball made of strips of silk, which was a favorite toy in ancient China.
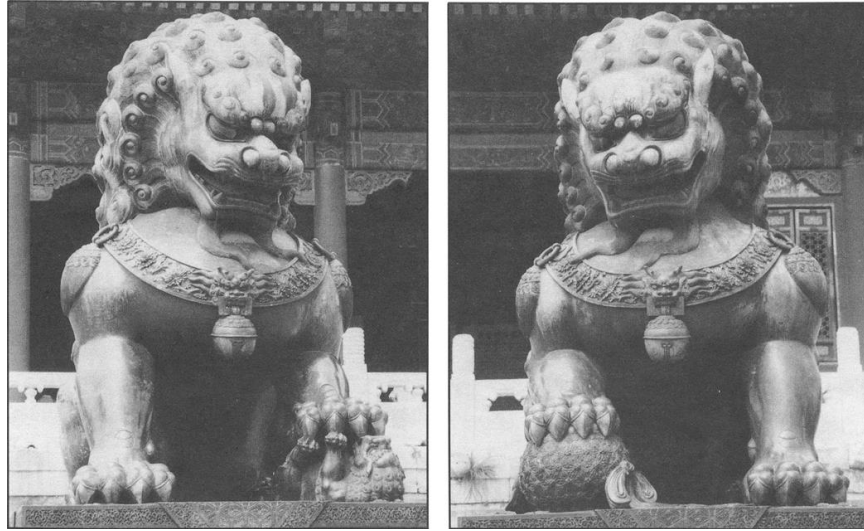
Figure 2. Two bronze lions in front of the *Gate of Supreme Harmony* (TAIHEMEN) in the Forbidden City, Beijing, China.
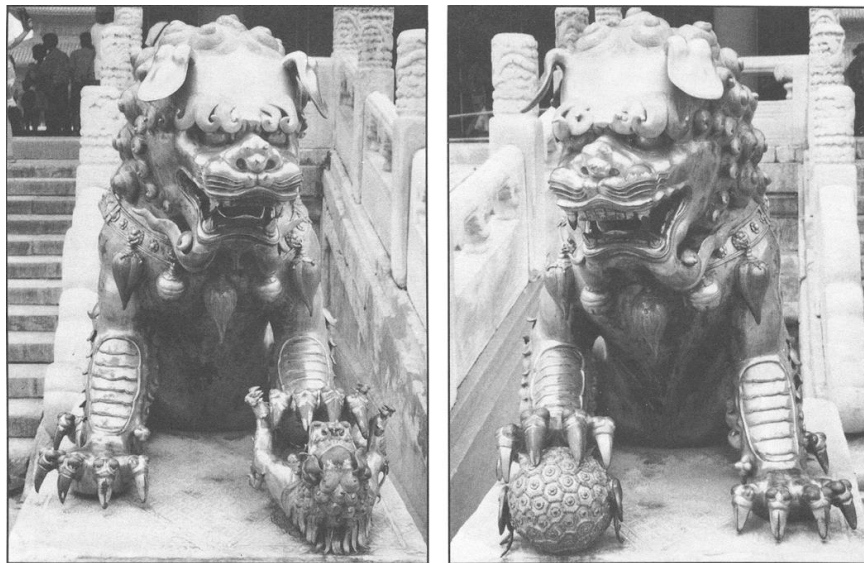

Figure 3. Two gold-plated lions in front of the *Gate of Heavenly Purity* (QIANQINGMEN) in the Forbidden City, Beijing, China.

Figure 2 shows a pair of bronze lions in front of the *Gate of Supreme Harmony* (Taihemen) in the Forbidden City, Beijing. It was made during the reign of the Ming Dynasty (1368-1644). An elaborate regular hexagonal decoration of the surface of the sphere is under the male lion's paw. It is not possible, however, to cover the surface of the sphere by a regular hexagonal pattern. Indeed, considerable chunks of the sphere are hidden by the lion's paw and by the stand itself on which the lion and the sphere stand. Other lions with similar decorations of the sphere are found in many other places.

An interesting pair of lions whose male partner has a sphere under the paw with a different decoration is shown in Figure 3, with sphere in a close-up in Figure 4. This pair

is in front of the *Gate of Heavenly Purity* (Qianqingmen) in the Forbidden City and dates back to the reign of Qian Long (1736-1796) of the Qing Dynasty. The surface of this sphere is decorated by a hexagonal pattern which, however, is interspersed by pentagonal shapes. Such a pattern can indeed cover the complete surface of a sphere.
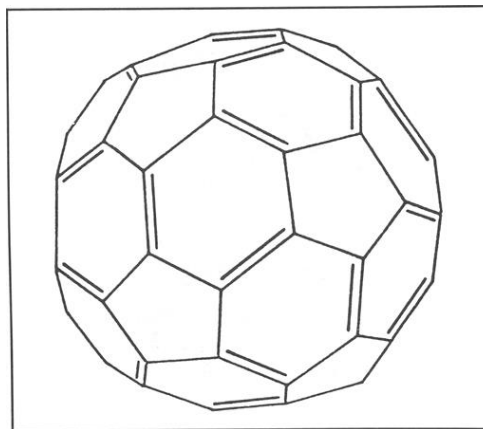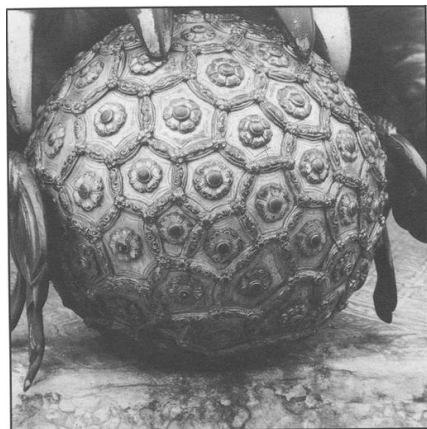
Left: Figure 4. Close-up of the sphere under the male lion's paw ; several pentagonal shapes are seen interspersed in the hexagonal pattern decorating the surface of the sphere.

Right: Figure 5. Structure [4] proposed for the super-stable C_{60} all-carbon molecule. Each vertex is occupied by a carbon atom. The single and double lines represent two different carbon-carbon linkages. This structure has been proved by a variety of physical and computational techniques (see, e.g., [5]).

Mathematicians have known, of course, that one can close an even-number of vertices with any number of hexagons (except one), provided 12 pentagons are included in the network (see, e.g., [1]). An important recent discovery in chemistry is related to such structures. When Kroto and co-workers observed [2] the great relative abundance of C_{60} molecules in their laser vaporization cluster-beam experiment, a search followed for the structure of this extraordinarily stable species. Kroto [3] describes eloquently how his previous encounters with Buckminster Fuller's work, and in particular the Geodesic Dome as the U.S. Exhibition Hall at the Montreal Expo, assisted him and his colleagues to arrive at the highly symmetrical truncated icosahedral structure (Figure 5). A visit to the Forbidden City might have been similarly instructive and beneficial.

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