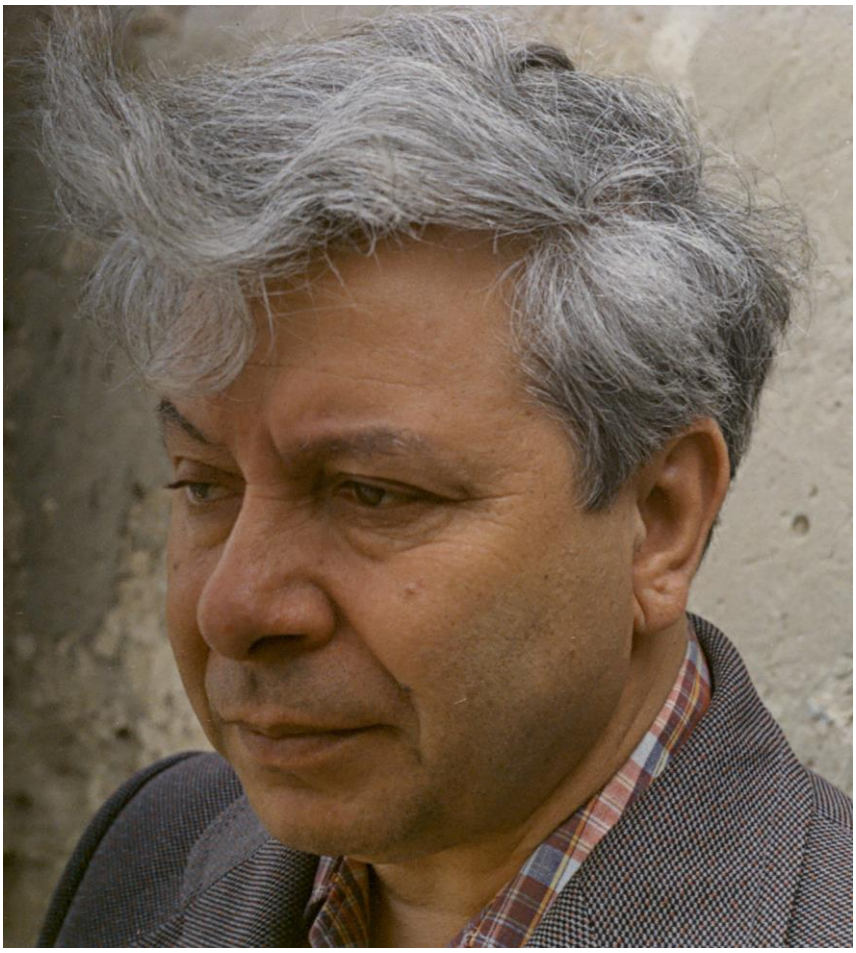

Khudu S. Mamedov (Mammadov) Azerbaijani crystallographer in Baku, 1982 (photograph by I. Hargittai). Mamedov often applied anti-symmetry in his periodic drawings and combined crystallography education and the preservation of cultural heritage.

Symmetry of Opposites: Antisymmetry[a]

István Hargittai and Magdolna Hargittai

Symmetry has long been identified with the properties of geometrical figures. The Russian crystallographer E. S. Fedorov, for example, gave it the following definition (quoted in [1]): "Symmetry is the property of geometrical figures of repeating their parts, or more precisely, their property of coinciding with their original position when in different positions." According to the Canadian geometer H. S. M. Coxeter [2], "when we say that a figure is 'symmetrical' we mean that there is a congruent transformation which leaves it unchanged as a whole, merely permuting its component elements."

However, it has also been recognized for a long time that symmetry, as observed in real nature, cannot be reduced entirely to this geometrical symmetry. K. Mislow and P. Bickart [3] observed in their epistemological note on chirality that "when one deals with natural phenomena, one enters 'a stage in logic in which we recognize the utility of imprecision.'" Material symmetry, devoid of the rigor of geometrical symmetry, has been viewed as applicable to material objects as well as abstractions with limitless implications [4]. Symmetry also connotes harmony of proportions, a rather vague notion according to Weyl [5]. Human ability to geometrize nongeometrical phenomena helps us see symmetry even in its "vague" and "fuzzy" variations [6, 7]. Thus Weyl [5] said Dürer "considered his canon of the human figure more as a standard from which to deviate than as a standard toward which to strive."

The vagueness and fuzziness of the broader interpretation of symmetry allow us to talk about degrees of symmetry. There must be a range of criteria, which may change from problem to problem, and may very well change in time as well. Today, Science is turning to the examination of the less orderly systems, yet symmetry considerations are not losing importance. On the contrary, their applications are gaining depth as well as breadth.

Chemistry [8], for example, is a science where the symmetry concept has played an increasing role, and not only in such areas as spectroscopy and crystallography but more recently even in such a seemingly nonexact field as organic synthesis. The so-called antisymmetry has become a seminal consideration in modern chemistry, for example, in the description of atomic and molecular orbitals of electronic structure and its changes and interactions during chemical reactions, and in the description of molecular vibrations.

"Operations of antisymmetry transform objects possessing two possible values of a given property from one value to the other" [9]. The simplest antisymmetry operation is color change. Let us first consider an identity operation and an antiidentity operation in Figure 1. Move on then to antireflection in Figure 2, and a few further examples of this in Figure 3. Of course, geometrical symmetries are not restricted to reflection, and Figure 4 presents examples combining color change with both reflection and rotation, after Shubnikov [1].

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[a] *The Mathematical Intelligencer* 1994, 16(2):60–66

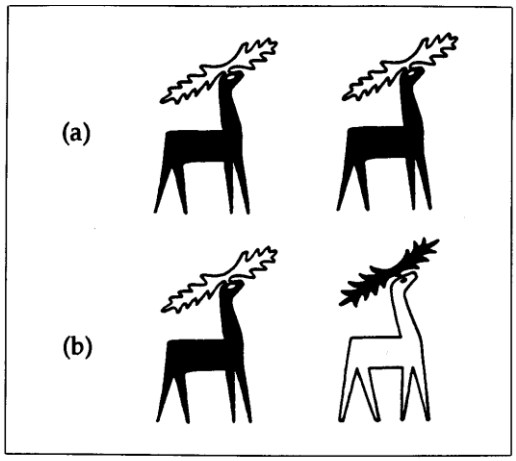


Figure 1. (a) Identity operation and (b) antiidentity operation [8].

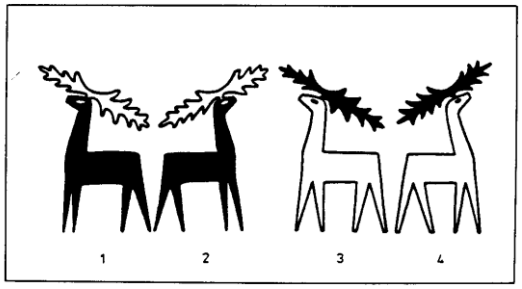


Figure 2. Reflections (1/2 and 3/4) and antireflections (1/4 and 2/3) [8].

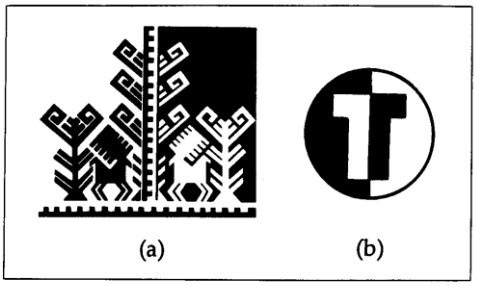


Figure 3. Antireflections: (a) Hungarian batik design; (b) logo of Tungsram Works.

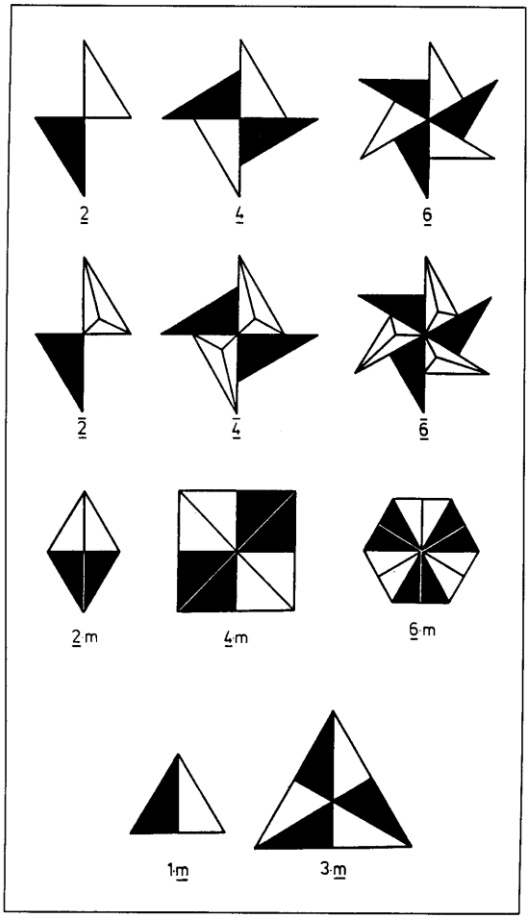


Figure 4. Antisymmetry operations after Shubnikov [1]. $\underline{2}$, $\underline{4}$, $\underline{6}$, antirotation axes; $\underline{\bar{2}}$, $\underline{\bar{4}}$, $\underline{\bar{6}}$, antireflection-rotation axes; $\underline{2} \cdot m$, $\underline{4} \cdot m$, $\underline{6} \cdot m$, antirotation axes combined with ordinary reflection planes; $1 \cdot \underline{m}$, $3 \cdot \underline{m}$, ordinary rotation axes combined with antireflection planes.

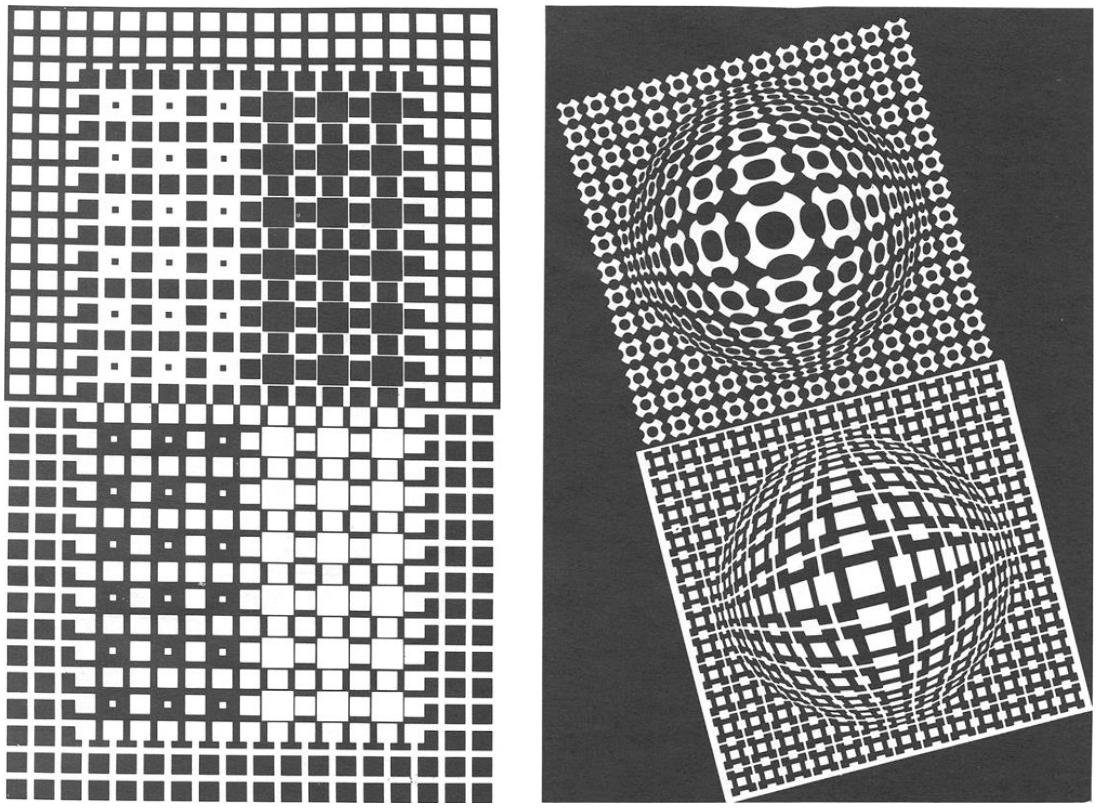


Figure 5. Pictures by Victor Vasarely (courtesy of the artist)

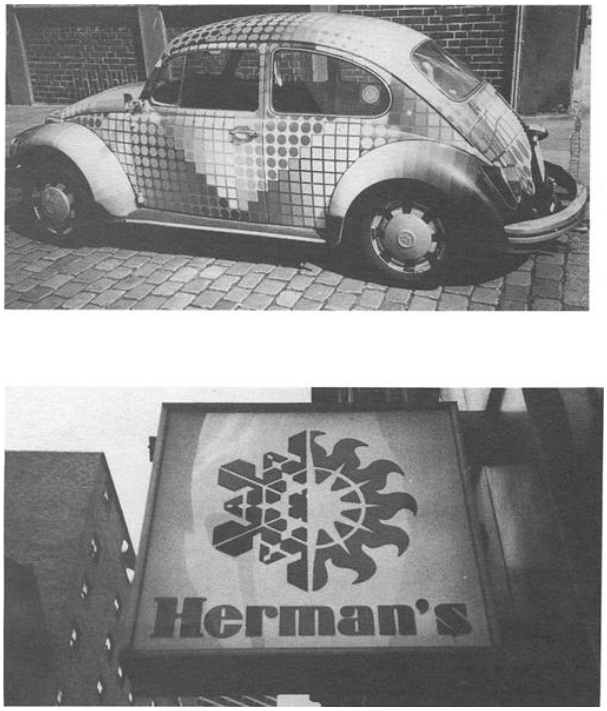


Figure 6. (top) Vasarely-like car decoration and (bottom) logo of sporting goods store in Boston, Massachusetts (photographs by the authors).

Figure 5 presents some op-art patterns. The first Vasarely picture, at least in this black-and-white version, illustrates simple color change between the upper and lower parts of the figure. The second Vasarely picture and the decorated car involve a change in the shape of the motifs in addition to the color change. Here we have alternative properties, color and shape, either of which can be changed into its opposite. We are also moving away from rigorous geometrical symmetry, and moving toward a wider application of the antisymmetry concept. The presence of a property turning into its opposite becomes the dominating effect; symmetry elements, such as reflection or rotation, may or may not accompany it.

Figure 6 shows the logo of a sporting goods store in Boston, Massachusetts. The antireflection plane relates winter and summer. Obviously, this store caters to both winter sports and summer sports fans. The color change in the self-serve/full-serve sign attracts attention in Figure 7, but the concepts may also be considered to have an antisymmetrical relationship.

Left: Figure 7. Self-serve versus full-serve gas station in Oahu, Hawaii (photograph by the authors). Right: Figure 8. "This is perestroika to some." An award-winning Soviet poster from 1987.

The Perestroika[b] poster of Figure 8 displays color change only, and the implication is ironic: Forces against reform would like to reduce the significance of Perestroika to mere color changes.

Let us interrupt our visual examples for two literary examples. The first refers to some antisymmetrical geographical relationships between, say, Western Europe and New Zealand. These locations can be connected by a straight diameter of the Earth going through its center. The noted American journalist James Reston [10] writes in his "Letter from Wellington. Search for the End of the Rainbow": "... Nothing is quite the same here. Summer is from December to March. It is warmer in the North Island and colder in the South Island. The people drive on the left rather than on the right. Even the sky is different--dark blue velvet with stars of the Southern Cross—and the fish love hooks." (He might have added, cyclones go clockwise there, as does water draining from a sink.)

The other example is taken from the Hungarian writer of the 1930s, Frigyes Karinthy, from a short story "Two diagnoses" [11]. The same person, Mr. Same, goes to see a physician at two different places on two different occasions. At the recruiting station he would obviously like to avoid getting drafted, whereas at the insurance company he

[b] The Russian word “Perestroika,” re-structuring, was a buzz word by Soviet President Mikhail Gorbachev when he tried to save the Soviet system by introducing reform.

would like to acquire the best possible terms for his policy. His answers to the identical questions of the physicians are related by antisymmetry.

At the recruiting station
Mr. Same: *Broken-looking, sad, ruined human wreckage, feeble masculinity, haggard eyes, unsteady movement.*
Physician: How old are you?
Mr. Same: Old... very old, indeed.
Physician: Your I.D. says you're thirty two.
Mr. Same: *With pain.* To be old is not to be far from the cradle--but near the coffin.
Physician: Are you ever dizzy?
Mr. Same: Don't mention dizziness, please, Doctor, or else I'll collapse at once. I always have to walk in the middle of the street, because if I look down from the curb, I become dizzy at once.

At the insurance company
Mr. Same: *Young athlete with straightened back, flashing eyes.*
Physician: How old are you?
Mr. Same: *Coyly*, Oh, my gosh, I'm almost ashamed of it... I'm so silly...
Physician: Your I.D. says you're thirty two.
Mr. Same: To be young is not to be near the cradle, but far from the coffin.
Physician: Are you ever dizzy?
Mr. Same: Quite often, sorry to say. Every time I'm aboard an airplane and it's upside-down, and breaking to pieces. Otherwise, not...

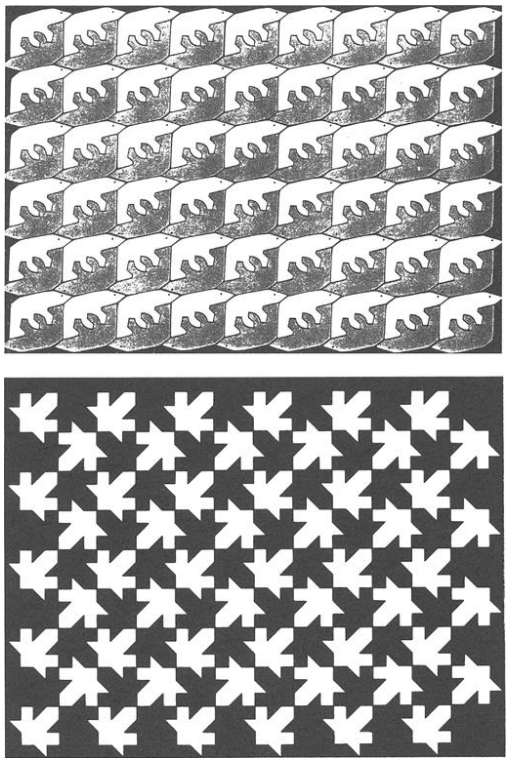
Figure 9. Belgian holiday ad in Flemish and French from 1983 (photograph by the authors).

Returning now to visual examples, Figure 9 shows a Belgian travel ad, and the changing property is the language, Flemish/French. The horizontal antireflection is very approximate in Figure 10 on the election poster by the Alliance of Young Democrats at the time of the 1990 Hungarian elections.[c] The Viennese dancing school ad (Figure 11) relates an elephant's legs and a girl's by antisymmetry, obviously for the ability to dance. Figure 12 shows two military jets and a sea gull off Bodø, Norway, a military base, and they may imply the polarity of war and peace.

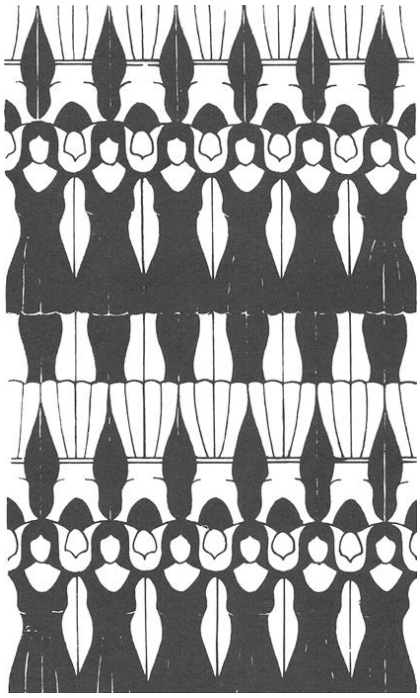
[c] On the poster, Leonid Brezhnev was the Soviet President and Erich Honecker the East German communist leader

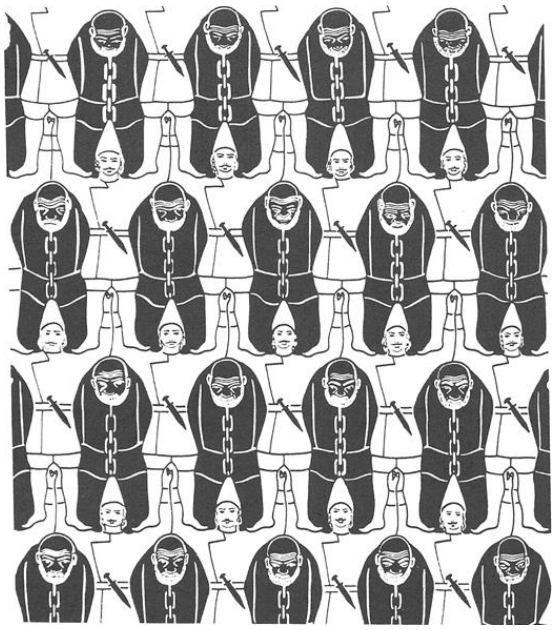
Figure 10. Election poster by the (Hungarian) Alliance of Young Democrats (FIDESz), 1990. Upper half: Brezhnev and Honecker. Text in the middle: "Please, make your choice."

Left: Figure 11. Viennese dancing school ad (photograph by the authors).
Right: Figure 12. Military jets and a sea gull, off Bodo, Norway, 1981 (photograph by the authors).



Left: Figure 13. E Brisse: (a) "Northwest Territories"; (b) "Canada" (From Ref. 13, reproduced by permission). Right: Figure 14. M. C. Escher: "Dogs" (From Ref. 16, reproduced by courtesy of the International Union of Crystallography).





Drawings by Kh. Mamedov, Left: Figure 15. "Girls," Right: Figure 16. "Unity" (From Ref. 17, courtesy of Professor Mamedov).

A few examples of translational antisymmetry are shown above. Apparently, the first systematic discussion of the 46 two-color two-dimensional patterns was communicated by H. J. Woods in 1936, in a work recently saved from oblivion by D. W. Crowe [12]. Woods pointedly called these two-color patterns “counterchange" patterns. The first two of our illustrations (Fig. 13) are by a Canadian crystallographer, F. Brisse [13]. In one, the polar bear is subjected to a twofold rotational antisymmetry and then translation in two directions. In the other, the two-dimensional space group of the pattern, disregarding color change, would be *p4gm*. This pattern has already been used by G. Po1ya [14] among his representations of the 17 two-dimensional space groups. It may also be found as a typical decoration in Islamic geometrical patterns [15]. However, in Brisse's pattern there is a two-color change during a complete revolution. There is then translation in two directions. Further simple color changes are involved in the next two figures. M. C. Escher's famous "Dogs" [16] is an excellent illustration of closest packing (Fig. 14). The color change is combined with glide lines. Reflection is also involved in generating Kh. Mamedov's “Girls” in Figure 15 [17]. The Azerbaijani crystallographer's other drawing "Unity" (Fig. 16) once again combines geometrical symmetries with a conceptual opposition: young versus old.

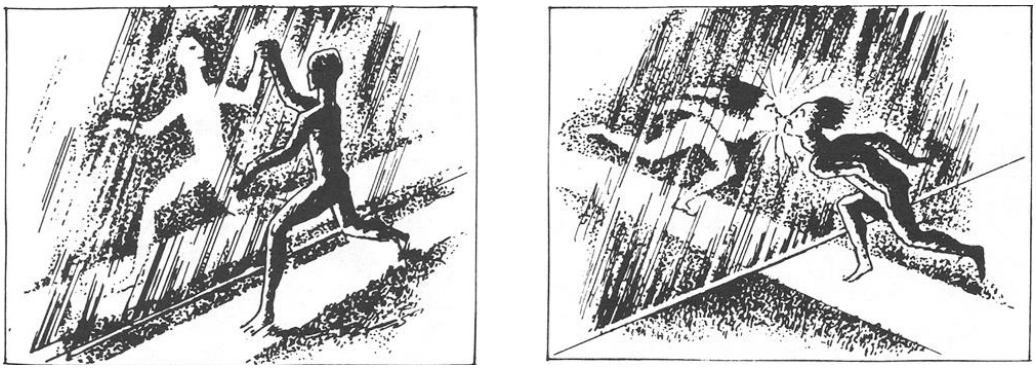


Figure 17. Symmetric (a, when the road chosen is parallel to the mirror) and antisymmetric (b, when we chose a road perpendicular to the mirror) consequences of reflection (drawing courtesy of architect G. Doczi [8]).

The symmetric and antisymmetric consequences of reflection for two movements are illustrated in Figure 17. Suppose we walk alongside a long wall of mirror (Fig. 17a). Our mirror image will be walking with us; the two velocities will be the same. Now walk from a distance toward the mirror, perpendicular to it (Fig. 17b). In this case, our mirror image will have a different velocity from ours. The speed will be the same again, but the direction will be the opposite. If we don't stop in time, we shall collide.

We conclude our discussion by mentioning A. Koestler's concept of bisociation. According to Koestler [18], the connection in thought association is made between thoughts on the same plane, whereas bisociation refers to connection of thoughts from different planes. Thus, bisociation may be considered to be the antisymmetric partner of thought association. Let us just quote one example from Koestler: "The Prince, travelling through his domain, noticed a man in the cheering crowd who bore a striking resemblance to himself. He beckoned him over and asked: 'Was your mother ever employed in my palace?' 'No Sire,'—the man replied.—'But my father was.'"

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