Symmetry by Numbers[1]

Mario Livio, The Equation that Couldn't be Solved: How Mathematical Genius Discovered the Language of Symmetry Simon & Schuster 2005, 368 pp

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The equation in the title is the quintic equation, the mathematical genius is Évariste Galois (1811–32), and the language of symmetry he discovered is group theory. Symmetry combines both beauty and science, and can easily be seen in the world around us. But before he could use it in science, Galois had to create the necessary mathematical tools. The world was slow to listen, and it took almost a hundred years for the practical value of group theory to be truly appreciated. Galois, meanwhile, was killed in a duel at an early age. In *The Equation that Couldn't be Solved*, Mario Livio follows his brief existence like a sleuth.

Born into a scholarly family in a Paris suburb in Napoleonic France, Galois was educated at home before being sent to a boarding school in Paris that rivaled the English schools of the time for austerity and rigid discipline. He was not a great success at school, but soon found satisfaction in mathematics, which became his sole occupation by the time he was 16. Having failed to gain entrance to a more prestigious college, he continued his studies in a high school.

Galois was still only 17 when he continued work started by the Norwegian mathematician Niels Henrik Abel, showing in general terms whether an equation is solvable by a formula or not. For this he introduced the seminal concept of a group, and created a new branch of algebra now known as Galois theory. His first publication appeared in 1829, but a combination of neglect and egotism prevented senior mathematicians of the day from giving him the exposure he deserved. When the work was finally introduced to the French Academy of Sciences, it was hardly appreciated.

Nonetheless, Galois continued his creative work, against all the odds. He failed another entrance examination as people greatly inferior to him could not appreciate his work, and lost his adored father, a Republican, who was driven to suicide by his royalist political opponents. Young Galois also had a passion for Republican revolution and served a prison term for his political activities. He fell in love with an undeserving girl and was killed in a duel that was related to this unfortunate entanglement. During the night before the tragedy, Galois hurriedly wrote a profound description of his group theory, remarking in the margin: "I have no time."

The Equation that Couldn't be Solved covers a remarkable number of different topics, including biographies of scientists and mathematicians. It also covers the Rubik cube and other puzzles; string theory; supersymmetry; the origin of creativity; the

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relationship between the external symmetry of the human face and body, and mate selection and sex life; and much more. Livio examines the contributions of others that led up to Galois' discovery, and gives a panoramic view of the direct, as well as quite remote, applications of group theory.

Very little escapes Livio's attention, especially in twentieth-century physics. But one omission is the contribution to the story of Eugene Wigner. He applied group theory to quantum mechanics in the 1920s, when most of his contemporaries were yet to value it: Wolfgang Pauli called it "die Gruppenpest" — roughly translated as "that pesky group business". Wigner was awarded a Nobel Prize in 1963 for this work.

Another omission from the book is that, in discussing crystallography, Livio stops at the classical notions of symmetry and defines crystallography as "the science studying the structures and properties of assemblies made of very large numbers of identical units". This idea supposes regularity and periodicity, and was largely a result of the tremendous success of X-ray diffraction in the twentieth century. Recently, however, the field has embraced other structures, such as the newly discovered quasi-crystals of regular but non-periodic patterns. It was an early suggestion by British crystallographer Alan Mackay that the rules describing 'crystal' structures be relaxed — and they have been. They now include structures that fall beyond the 230 space groups, and the new rules do not necessarily form groups.

Overextending the inferences from symmetry can be restrictive. As the historian of mathematics E. T. Bell said: "The cowboys have a way of trussing up a steer or a pugnacious bronco, which fixes the brute so that it can neither move nor think. This is the hogtie and it is what Euclid did to geometry."

The book seems a little biased in places when it emphasizes the omnipresence of symmetry, but it nevertheless makes a lively and fascinating read for a broad audience.